# ACM SIGGRAPH@UIUC

## Fast Image Convolutions

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# Image Convolution

- Traditionally, image convolution is performed by what is called the sliding window approach.
  - For each pixel in the image, a local neighborhood of pixels is multiplied by a weighting kernel, and then added up to get the value of the new pixel.
- In the most general case, the convolution of a 2D image is $O(m^2*n^2)$, where m is the width and height of the image, and n is the width and height of the convolution kernel.
  - This is quadratic in terms of image dimension and quadratic in terms of kernel dimension!

# Naïve Implementation

```
for (each pixel location p, in the old image)
{
  accumulation = 0
  weightsum = 0
  for (each pixel k, in the neighborhood of p)
  {
      accumulation += k*weight(k)
      weightsum += weight(k)
  }

  p, in new image = accumulation/weightsum
}
```

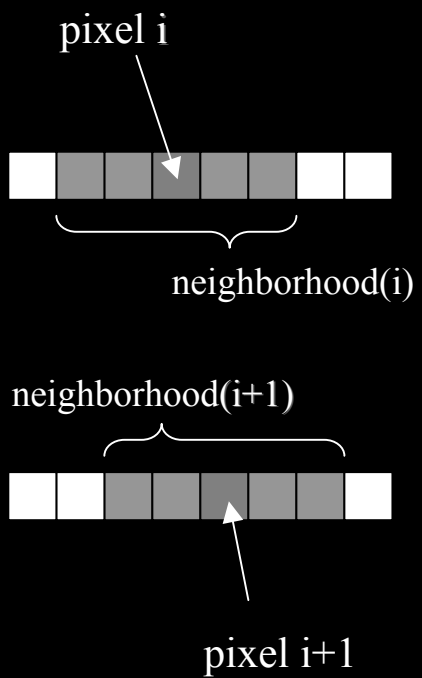
# The Weighting Kernel

- The weighting kernel is what will determine the properties of the convolution.
- For a simple box blur, all the weights are 1.
- For a Gaussian blur, the weights fall off according to a normal distribution away from the center of the kernel.
- A simple sharpen filter would have negative weights at nearby pixels, but a positive weight at the center.

# Speeding Up a Box Blur

- Instead doing a 2D box blur on an image, you can first do a horizontal motion blur, and then a vertical motion blur.
- This actually create an image that is equivalent to a box blurred image! Try it!
- This speeds it up from $O(m^2*n^2)$ to $O(m^2*2n)$!
- Quite a good speed-up, but we can still do better!

# Speeding Up a Box Blur Part 2



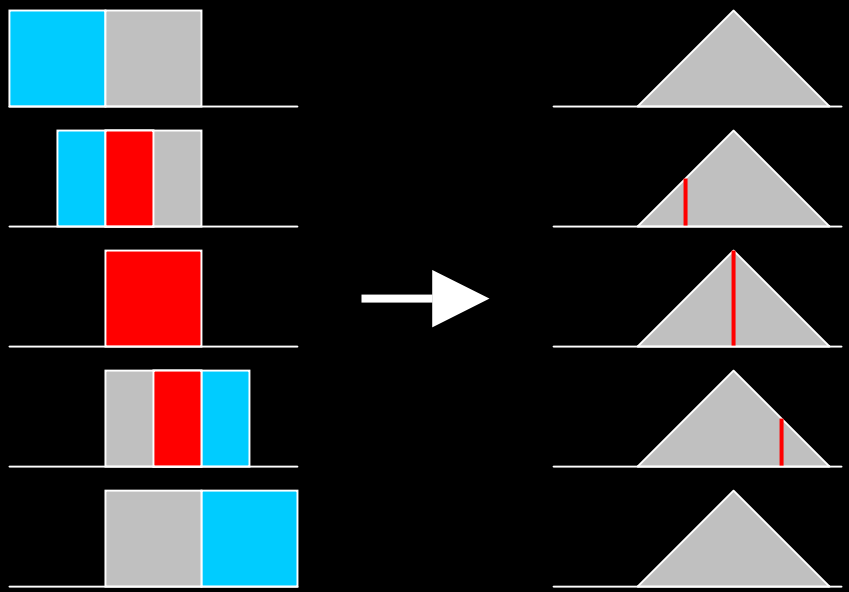

- Since we are now just doing 2 motion blurs, lets just consider motion blur:
- The accumulation of the neighborhood of pixel i, shares a lot of pixels in common with the accumulation for pixel i+1.
- In fact:
  - `accumulation(i+1) = accumulation(i) - leftmost pixel of neighborhood(i) + leftmost pixel of neighborhood(i+1)`
- This means we need to compute the whole kernel for only the first pixel in each row. Successive pixel blur values can be attained with just an add and a subtract to the previous blur value!
- Now its $O(m^2)$, only dependent on image resolution! Independent of blur size!

# What's next?

- We now have a box blur that is independent of blur width. What else could we ask for?
- Well, the box filter is not a very good blur kernel, we would like to use some better convolution kernels.
- How can we apply these concepts to a Gaussian blur kernel for instance?
- In order to answer this, lets first review some of the math behind convolutions …

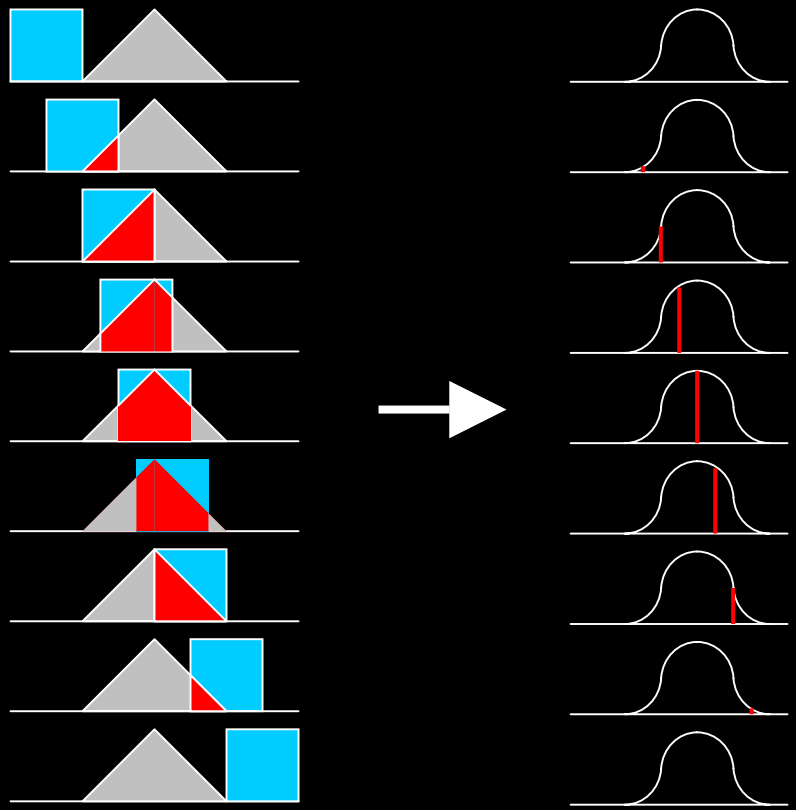
# The 1D case

- Lets look at convolution in 1D for simplicity.
- The sliding window is an intuitive way to visualize convolution.
- Convolution of two square waves (box filters) yields a triangle wave (tent filter, piecewise linear).



# The 1D case

- Convolution of a box filter (piecewise constant) with a tent filter (piecewise linear) yields a piecewise quadratic filter.



# Taking it Further

- The pattern continues. Box filtering the piecewise quadratic curve from the last slide will yield a piecewise cubic (Bernstein polynomials, NURBS).
- Each time we make the curve more “smooth.”
- Taking this to the limit will produce a Gaussian distribution.

# The Implications

- How does this relate to our 2D image blurs?
- We can put our fast box blur function to use in order to approximate a Gaussian blur!
- Applying our box filter two times will produce a tent filter, three times a piecewise quadratic, four times…
- We can therefore create a good approximation to a Gaussian blur that is still independent of radius! Its only dependent on the image size and the number of iterations we apply the box blur.

# 2D Filters

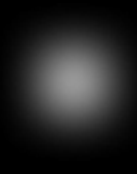


Motion Blur

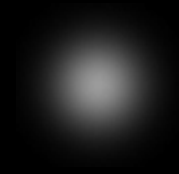
Box Blur

Tent Blur
(box 2 twice)

Piecewise Quadratic
(box blur 3 times)

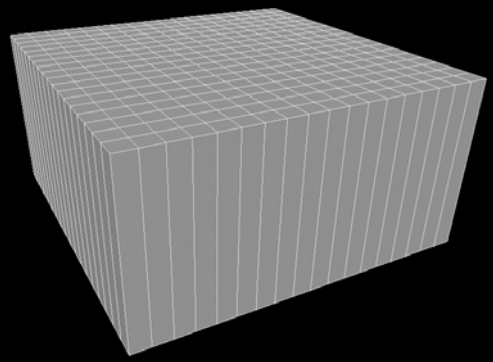


Piecewise hexic?
(box blur 6 times)

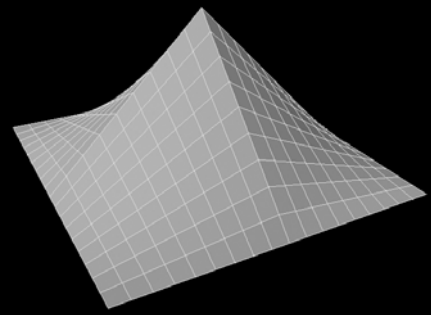


Gaussian Blur

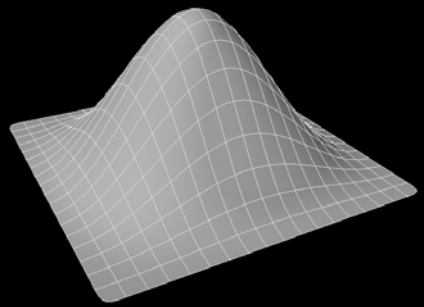
# 3D Visualization of 2D Filters



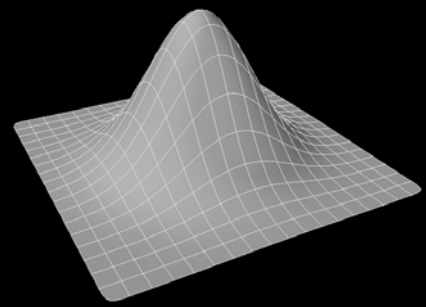
Piecewise Constant (Box) Filter



Piecewise Linear (Tent) Filter



Piecewise Quadratic Filter



Gaussian Filter

# Another Way

- Although it is still constant in time relative to the radius, to create a very smooth Gaussian approximation, many iterations are needed with this approach. In situations that a very nice blur is required, another method might be preferable.
- Our second speedup doesn't seem possible with anything but a box filter. The box filter was unique in that all its weights were equal, and that allowed us to just add a value and subtract a value to the accumulation for each pixel location.
- However, it turns out that our first speedup, doing two motion blurs, will work with other kernels as well!

# Other Kernels

- A problem arises, however, with the axis aligned nature of doing just two motion blurs. Using a tent filter for each motion blur will not create a nice radial tent filter (cone filter), but a normal, axis aligned tent. The same goes for any other kernel, it will have distinct axis aligned artifacts…
- With the exception of a Gaussian! The Gaussian has the unique quality of being the same whether it is defined along the radius, or along the X and Y axes separate.
- This means that if we do two Gaussian weighted motion blurs, this will create a *radially* symmetrical Gaussian kernel!

# Wrap Up

- Following these simple tips you can create image convolution routines that are orders of magnitude faster than the naïve implementations.
- Another thing to keep in mind is to pre-compute expensive kernel.
  - If you create a fast Gaussian blur function using two successive motion blurs, but you evaluate the Gaussian function every time you need to figure out a weight, you will see very little speedup.
  - Pre-compute the kernel! With our method, a radius 10 blur, would only require pre-computing/storing 11 values for the weights, since we are doing it in 1D each time.